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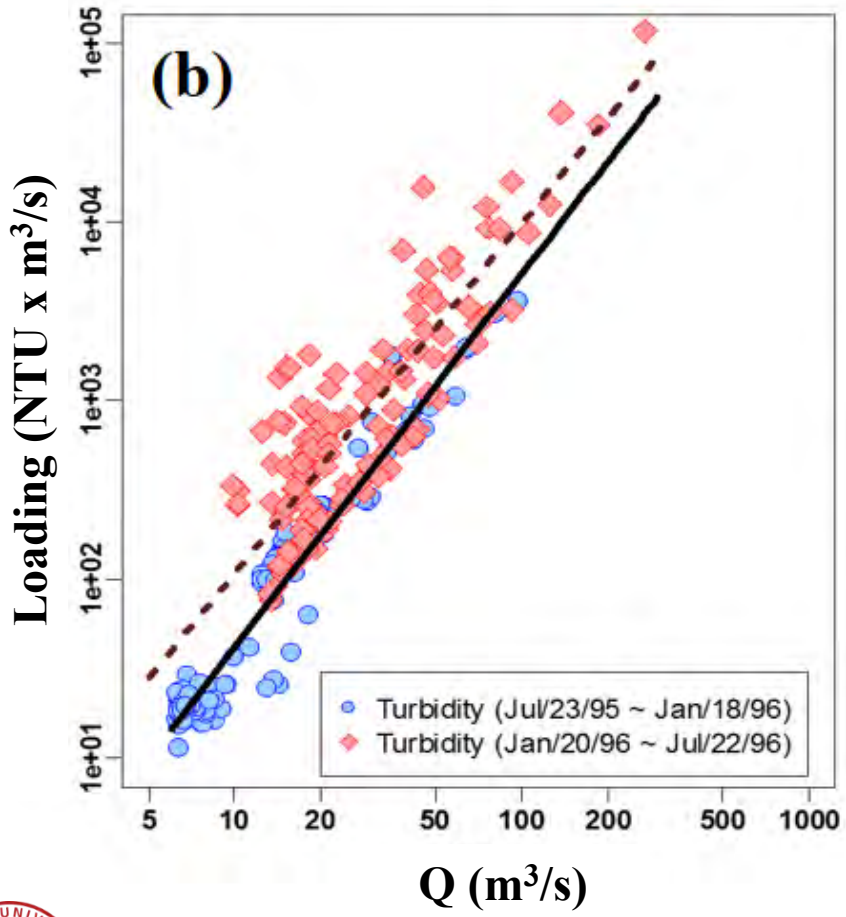
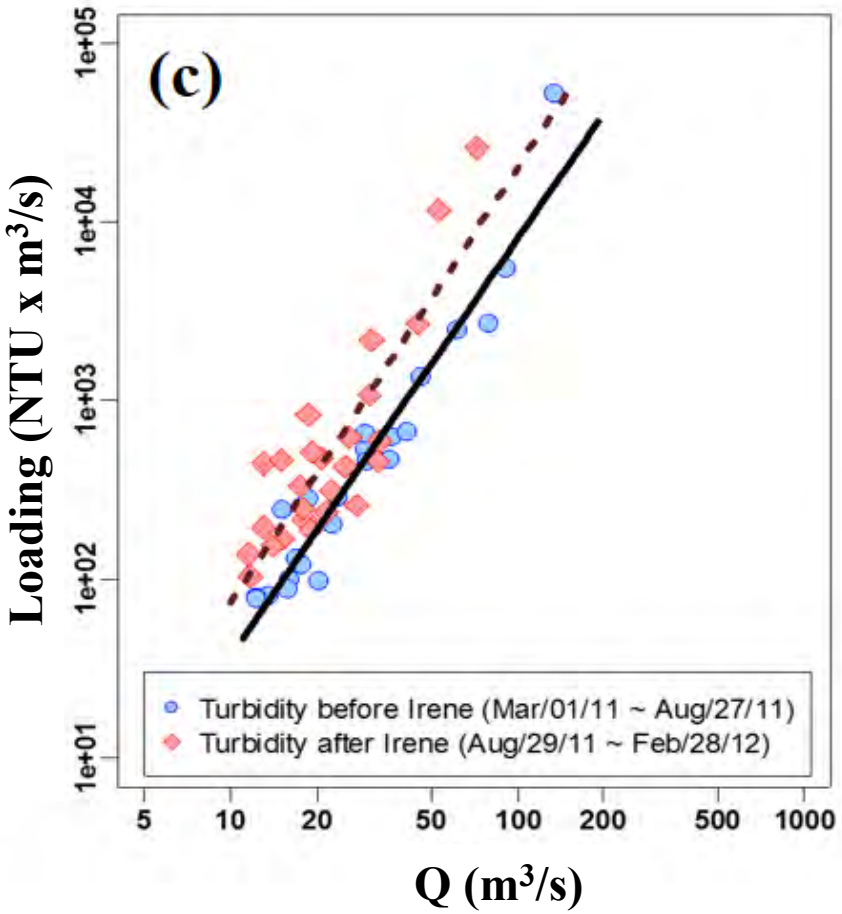
Time-varying suspended sediment-discharge rating curves to estimate climate impacts on fluvial sediment transport in the Esopus Watershed

Introduction

$$\log Tn_t = \beta_0 + \beta_1 \times \log Q_t + \varepsilon_t$$





Objectives of the presentation

1. Understand how the Esopus Creek rating curve has evolved over time

2. Model the underlying causes of time-varying rating curve behavior to support long-term simulation studies

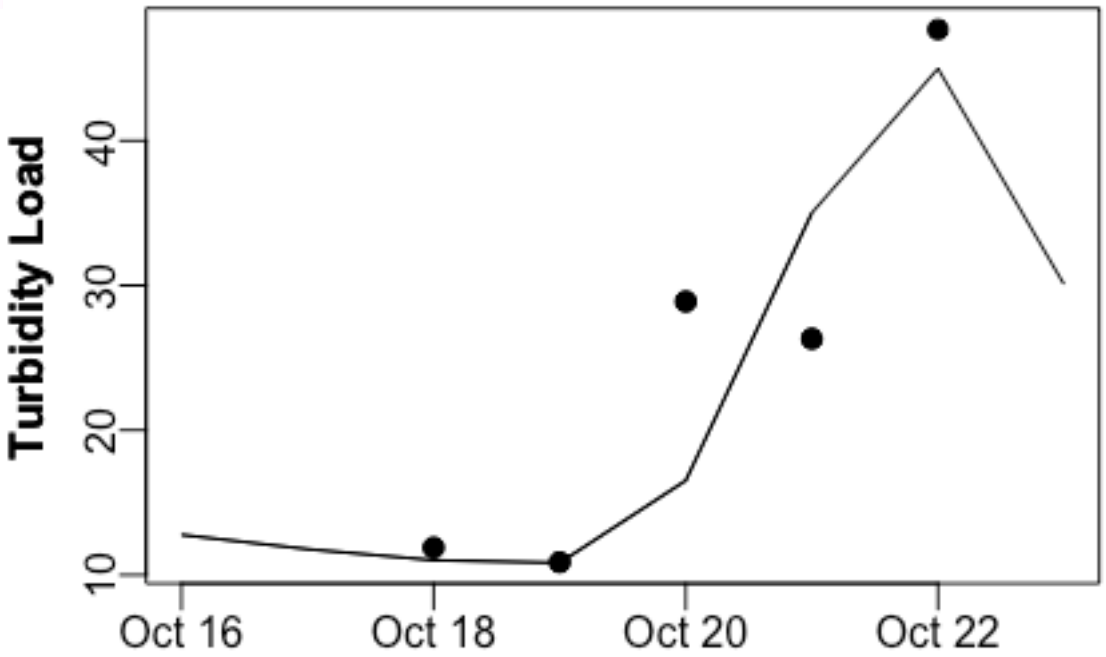
Objective 1

☐ *Understand how rating curve has varied in the past (Dynamic Linear Models)*

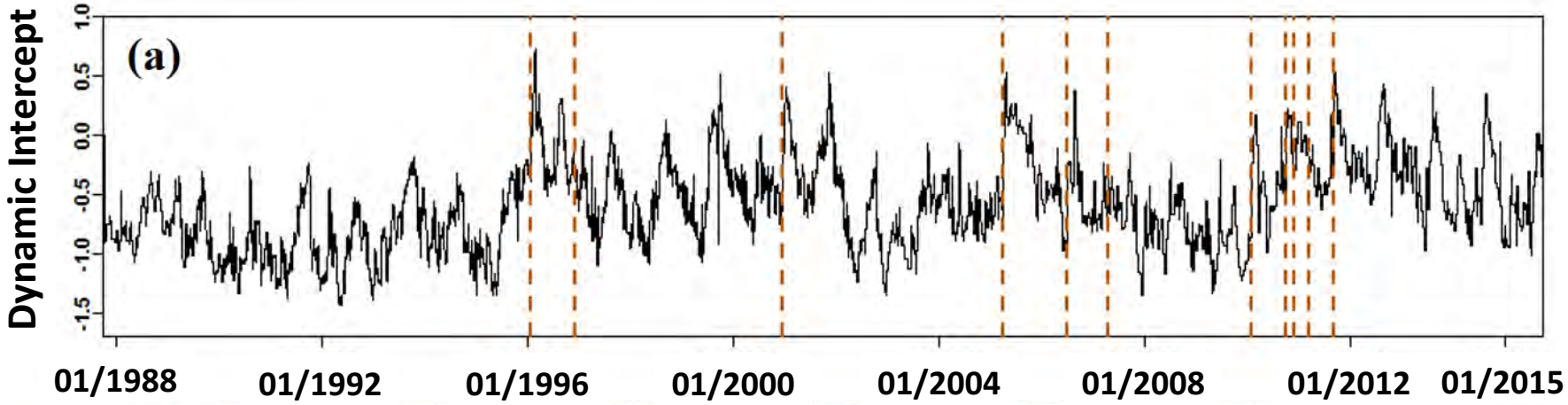
$$\log Tn_t = \beta_{0_t} + \beta_1 \log Q_t + \varepsilon_t \qquad \varepsilon_t \sim \mathcal{N}(0, {\sigma_\varepsilon}^2)$$

Coefficient is a random walk

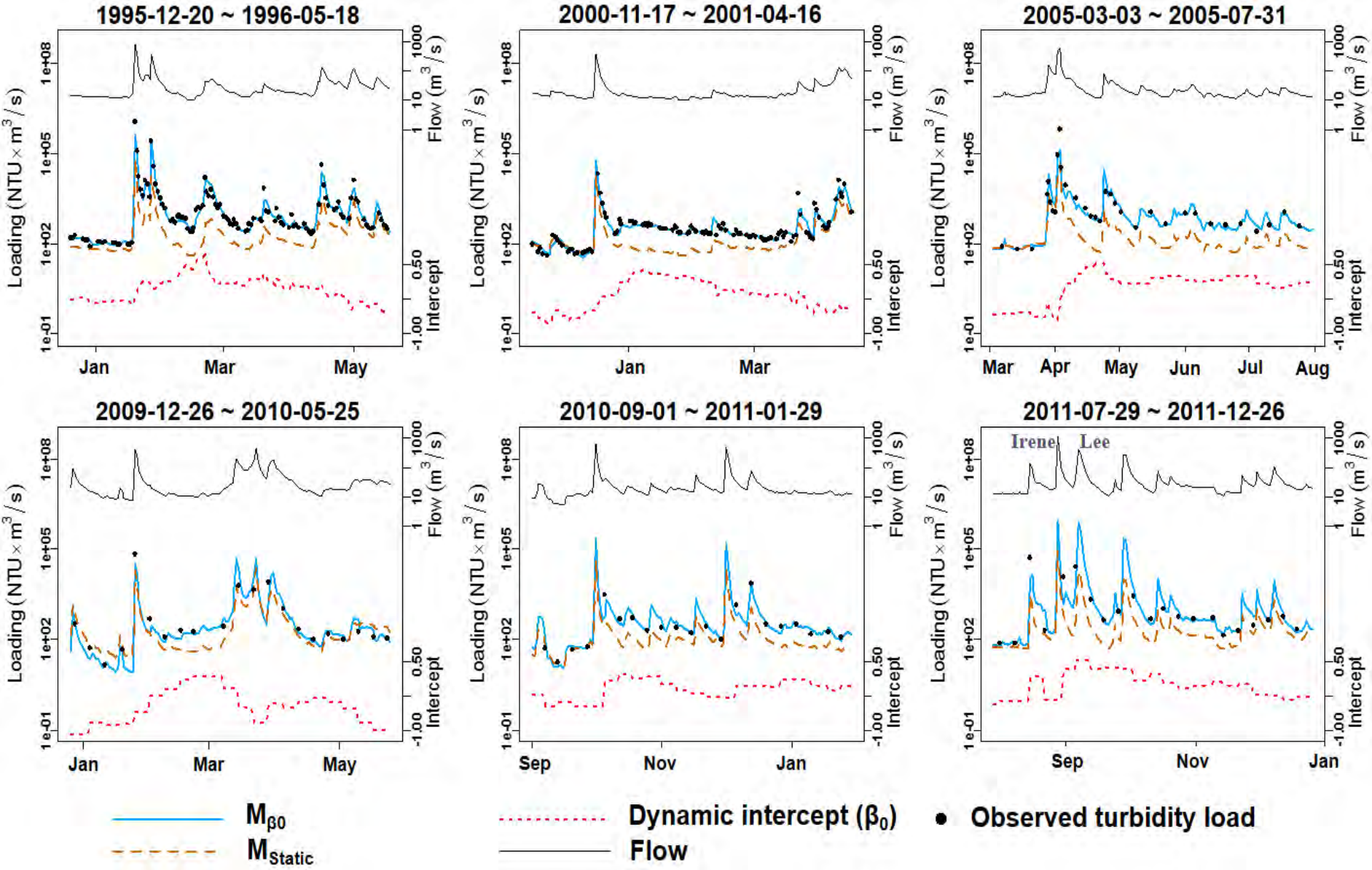
$$\beta_{0_t} = \beta_{0_{t-1}} + w_t \qquad w_t \sim \mathcal{N}(0, {\sigma_w}^2)$$



Time-varying intercept



Extreme floods



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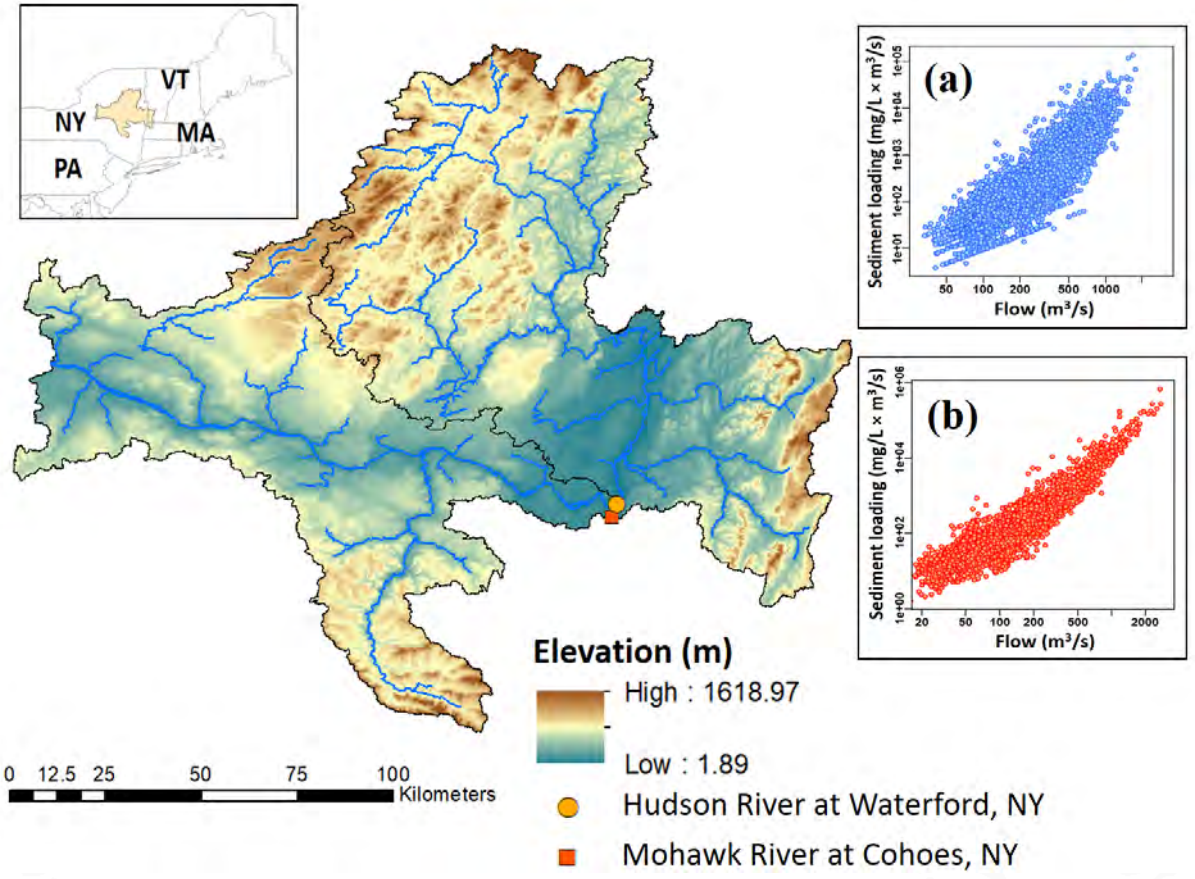
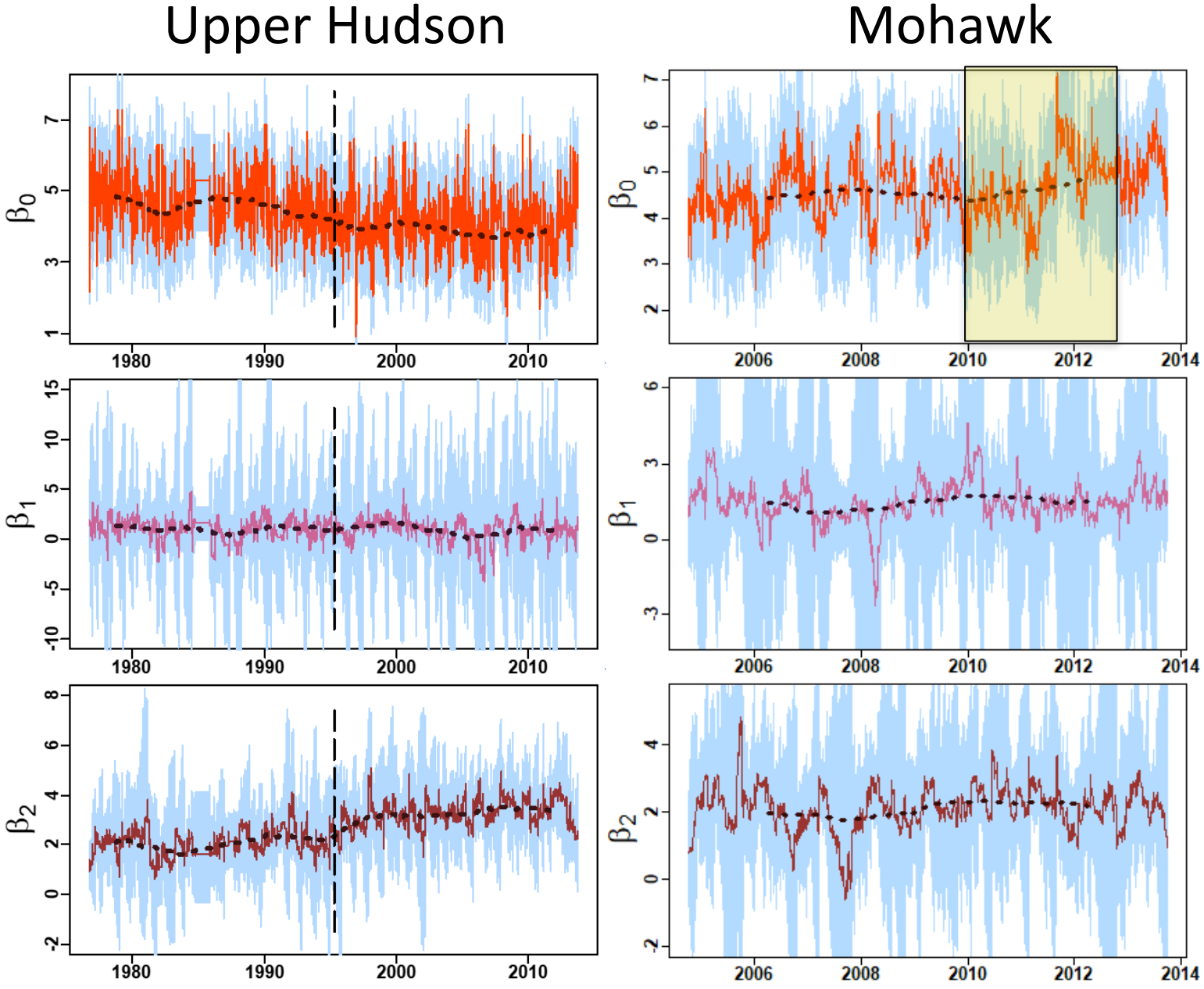
Expanding outside the Esopus

Particle filters for dynamic *non-linear* modeling



$$\log S_t = \begin{cases} \beta_0 + \beta_1 \log \widetilde{Q_t} + \varepsilon_{1,t} & \forall\ \widetilde{Q_t} < \widetilde{Q^*} \\ \beta_0 + \beta_1 \log \widetilde{Q^*} + \beta_2 (\log \widetilde{Q_t} - \log \widetilde{Q^*}) + \varepsilon_{2,t} & \forall\ \widetilde{Q_t} \geq \widetilde{Q^*} \end{cases}$$

Expanding outside the Esopus



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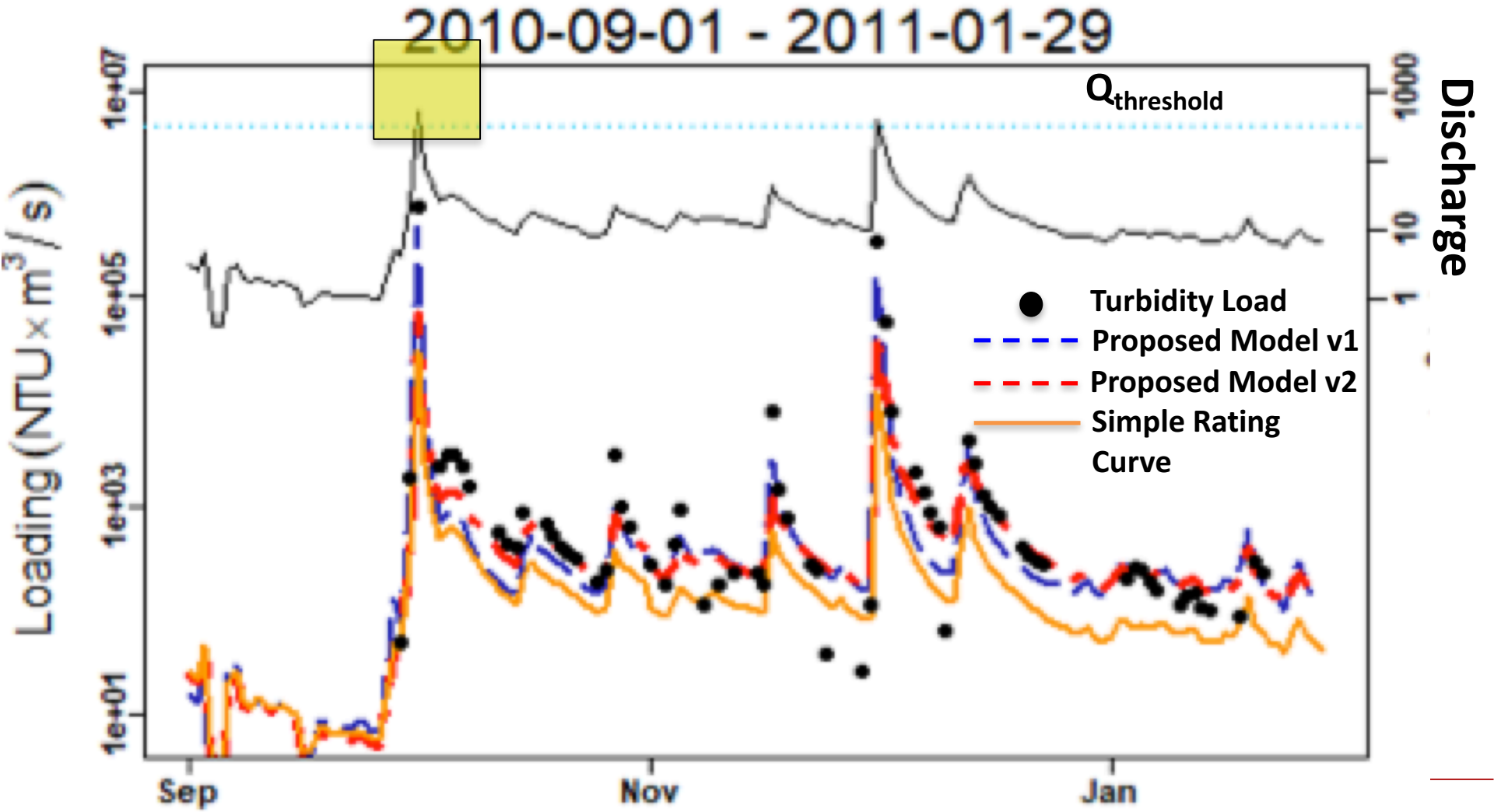
Objective 2

☐ *Develop a time-varying rating curve model for future*

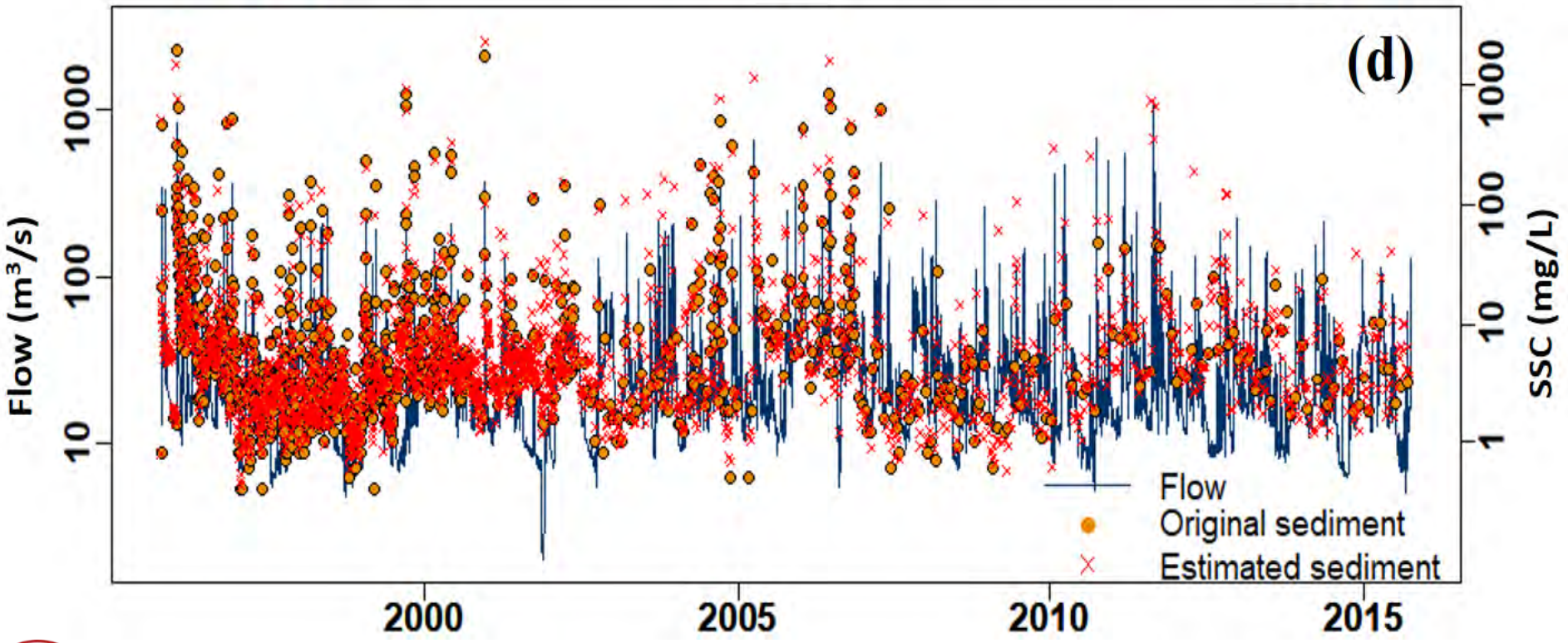


Methodology

Dynamic Linear Model (DLM) for SSC-Tn Relationship

$$\log SSC_t = \theta_{0,t} + \theta_1 log T_{n_t} + \varepsilon_t \qquad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

$$\theta_{0,t} = \theta_{0,t-1} + w_t \qquad w_t \sim \mathcal{N}(0, \sigma_w^2)$$



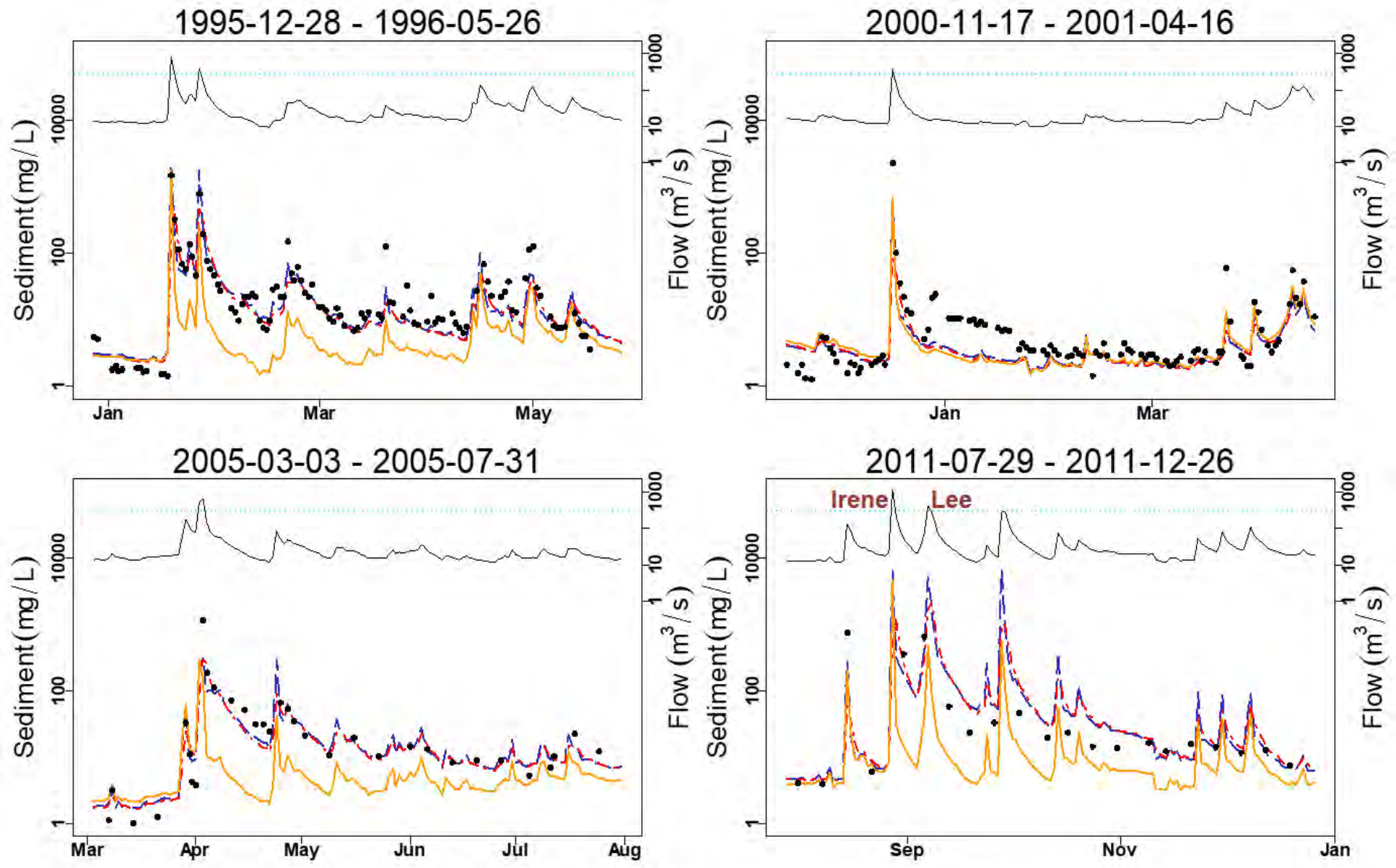
Inter-Model Comparison

Model	Model Equation	Time-varying Intercept
M_1	$\log rSSC_t = \beta_0 + \beta_1 logQ_{w_t} + \beta_2 \sin(2\pi i_t) + \beta_3 \cos(2\pi i_t) + \varepsilon_t$	No
M_2	$\log rSSC_t = \beta_0 + \beta_1 logQ_w + \beta_2 \sin(2\pi i_t) + \beta_3 \cos(2\pi i_t) + \beta_4 \log \widehat{Q_{s_{t-1}}} + \varepsilon_t$	No
M_3	$\log rSSC_t = \beta_0 + \beta_1 logQ_w + \beta_2 \sin(2\pi i_t) + \beta_3 \cos(2\pi i_t) + \beta_5 \frac{dlogQ_{w_t}}{dt} + \varepsilon_t$	No
M_4	$\log rSSC_t = \beta_0 + \beta^*_{0,t-1} + \beta_1 logQ_{w_t} + \beta_2 \sin(2\pi i_t) + \beta_3 \cos(2\pi i_t) + \varepsilon_t$	Yes
M_5	$\log rSSC_t = \beta_0 + \beta^*_{0,t-1} + \beta_1 logQ_{w_t} + \beta_2 \sin(2\pi i_t) + \beta_3 \cos(2\pi i_t) + \beta_4 \log \widehat{Q_{s_{t-1}}} + \varepsilon_t$	Yes
M_6	$\log rSSC_t = \beta_0 + \beta^*_{0,t-1} + \beta_1 logQ_{w_t} + \beta_2 \sin(2\pi i_t) + \beta_3 \cos(2\pi i_t) + \beta_5 \frac{dlogQ_{w_t}}{dt} + \varepsilon_t$	Yes

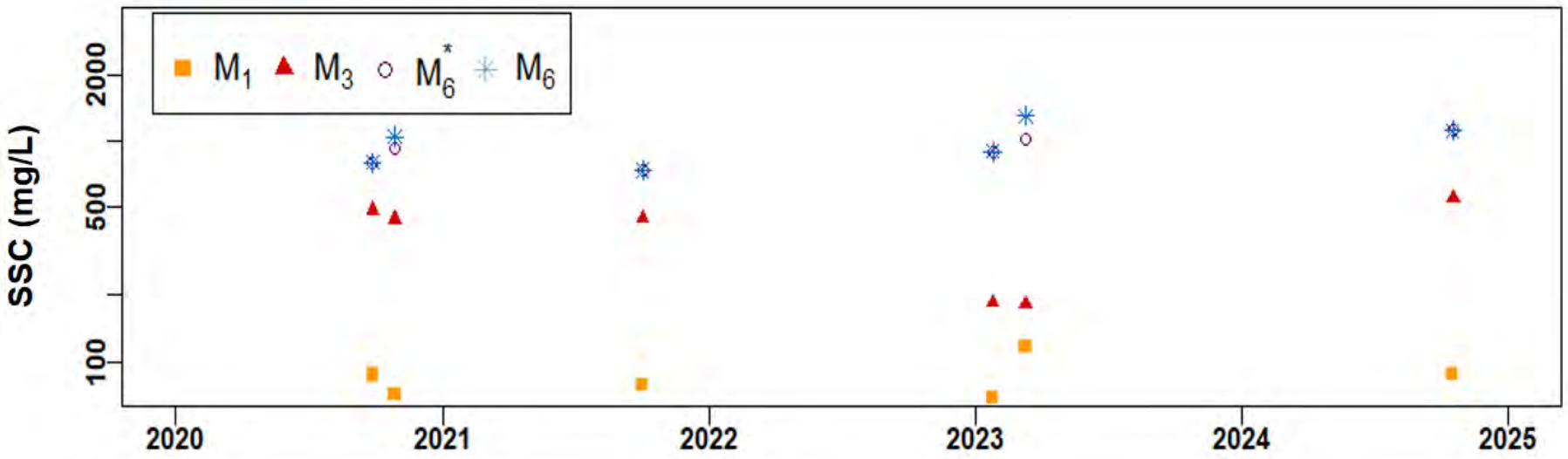
$$\beta^*_{0,t} = \varphi_1 \beta^*_{0,t-1} + \varphi_{2,t}\left(logQ_{w_t} - logQ_{threshold}\right)$$

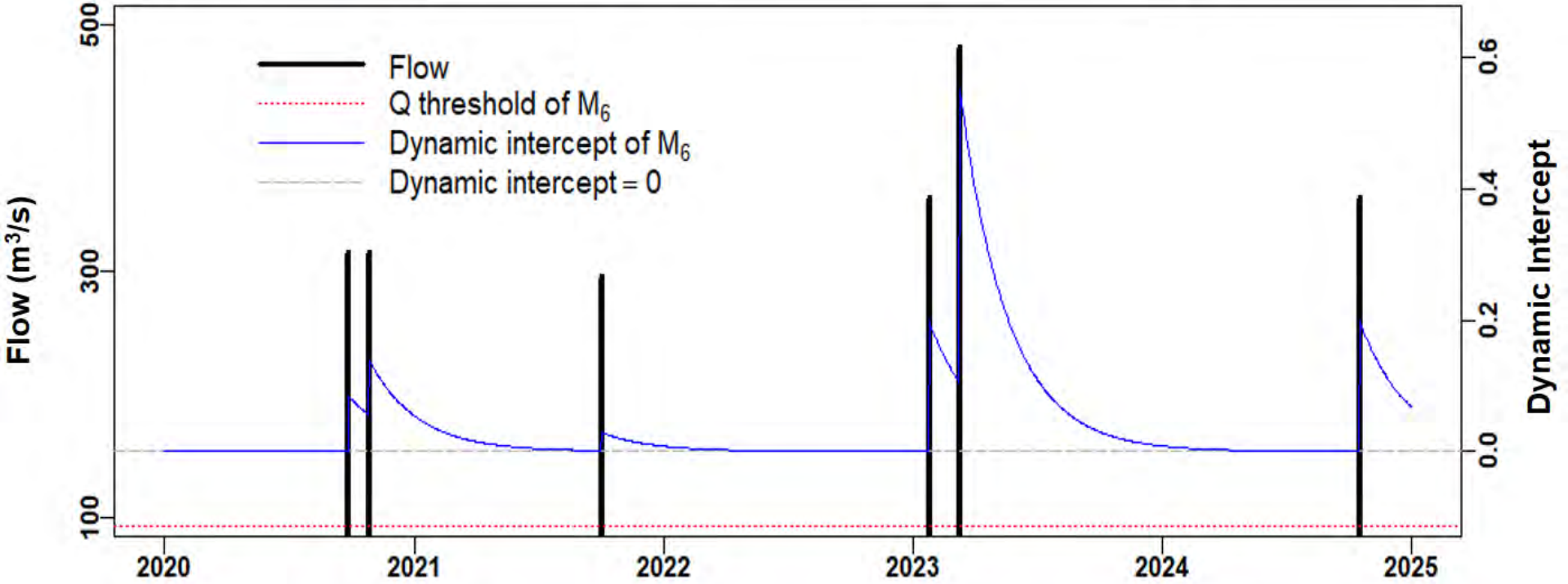
$$\varphi_{2,t} = \begin{cases} 0 & \forall\, Q_{w_t} < Q_{threshold} \\ \gamma & \forall\, Q_{w_t} \geq Q_{threshold} \end{cases}$$

Performance During and After Extreme Events

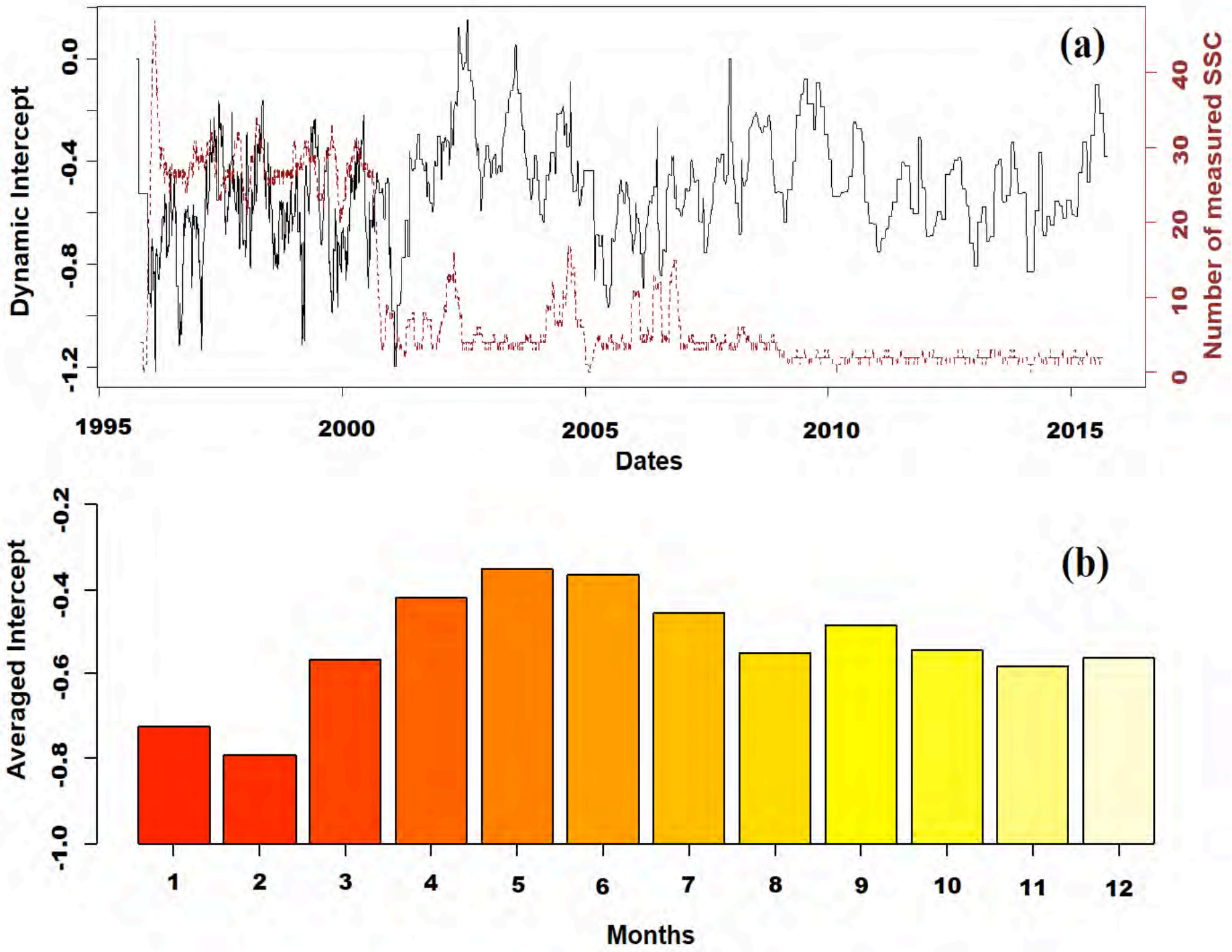


Peaks-Over-Threshold

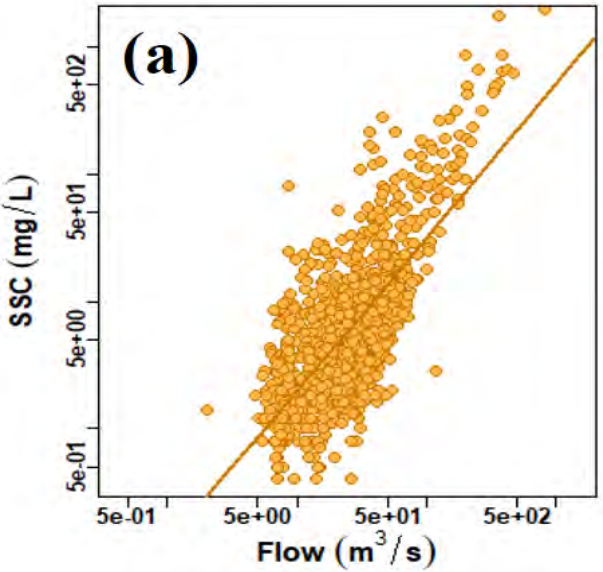


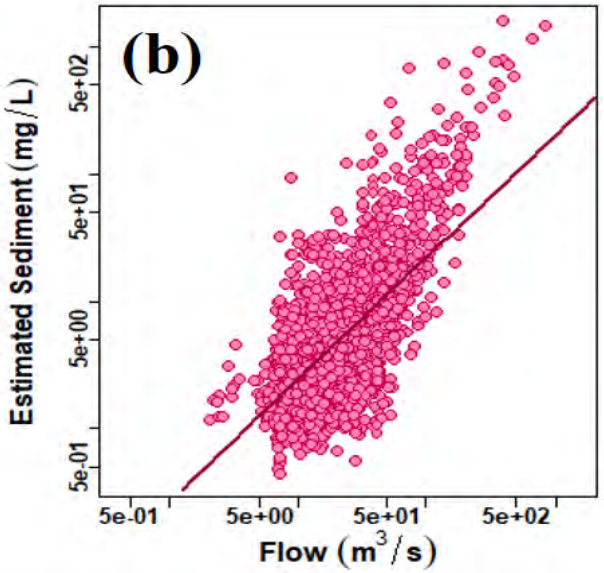


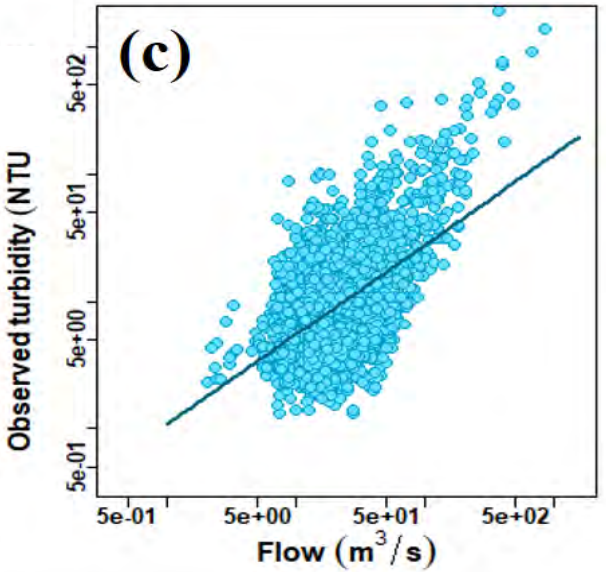
Thank You

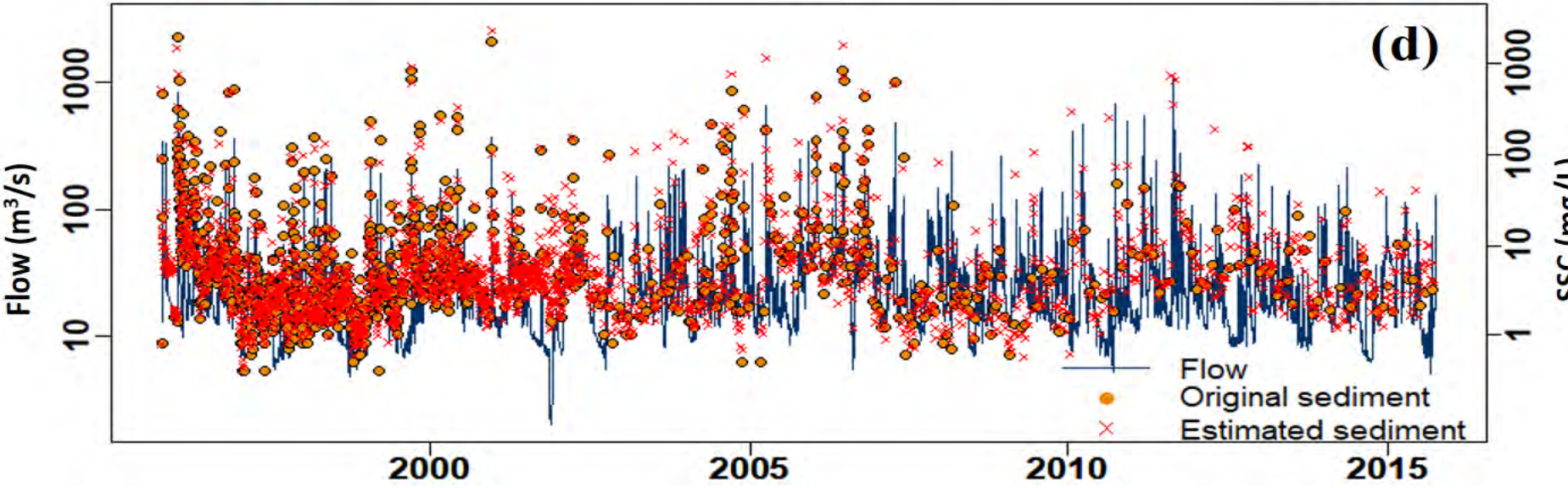

(a)
Dynamic Intercept
0.0
-0.4
-0.8
-1.2
Number of measured SSC
0
10
20
30
40
1995
2000
2005
2010
2015
Dates
(b)
Averaged Intercept
-0.2
-0.4
-0.6
-0.8
-1.0
1
2
3
4
5
6
7
8
9
10
11
12
Months

Validating the DLM for SSC-Tn









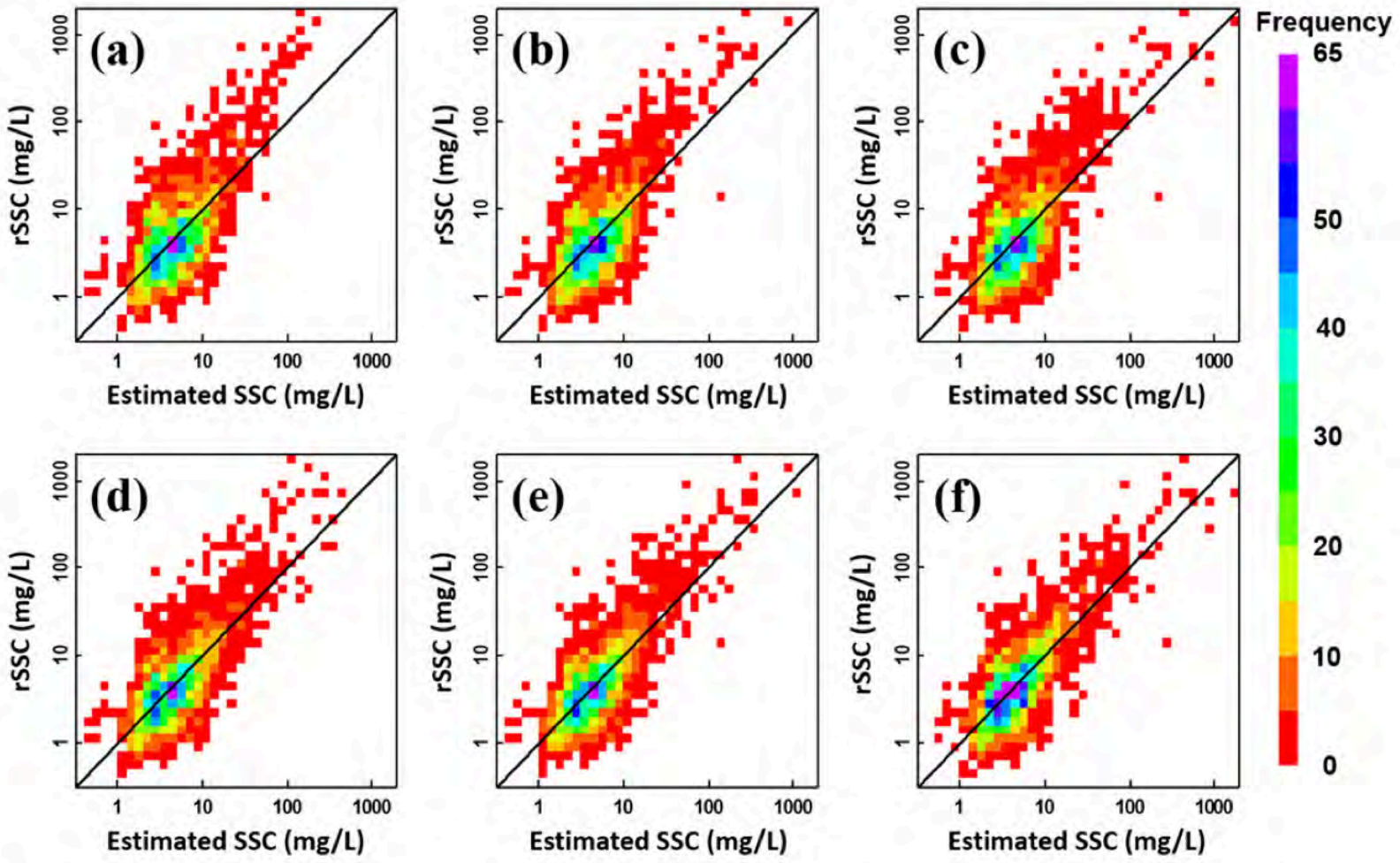


FIGURE 5 Two dimensional histograms between reconstructed suspended-sediment concentration (rSSC) and modelled suspended-sediment concentration (SSC) estimated by (a) M_1, (b) M_2, (c) M_3, (d) M_4, (e) M_5, and (f) M_6

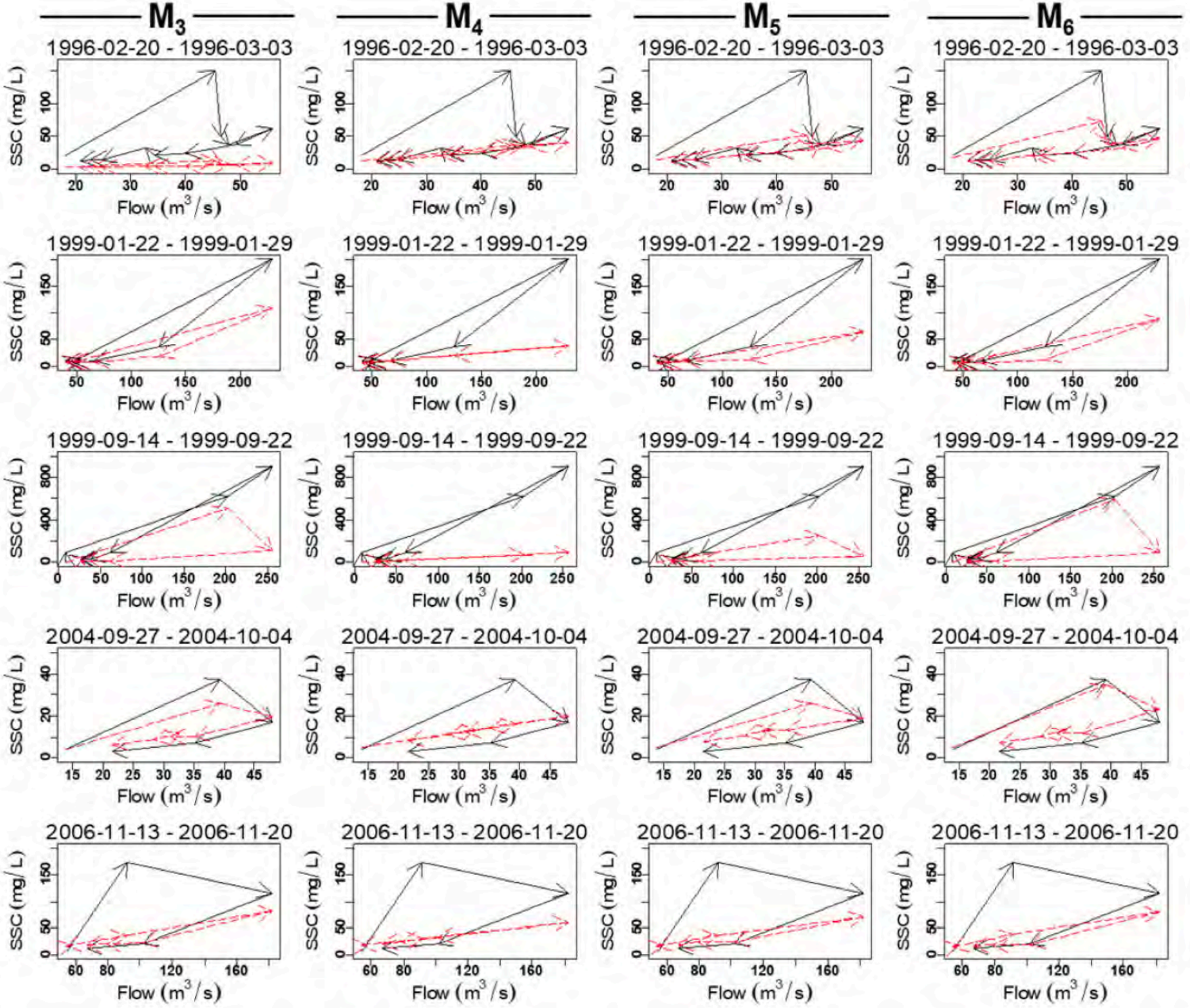


FIGURE 6 Hysteresis loops for model estimated suspended-sediment concentration (SSC; red) and reconstructed suspended-sediment concentration (black)

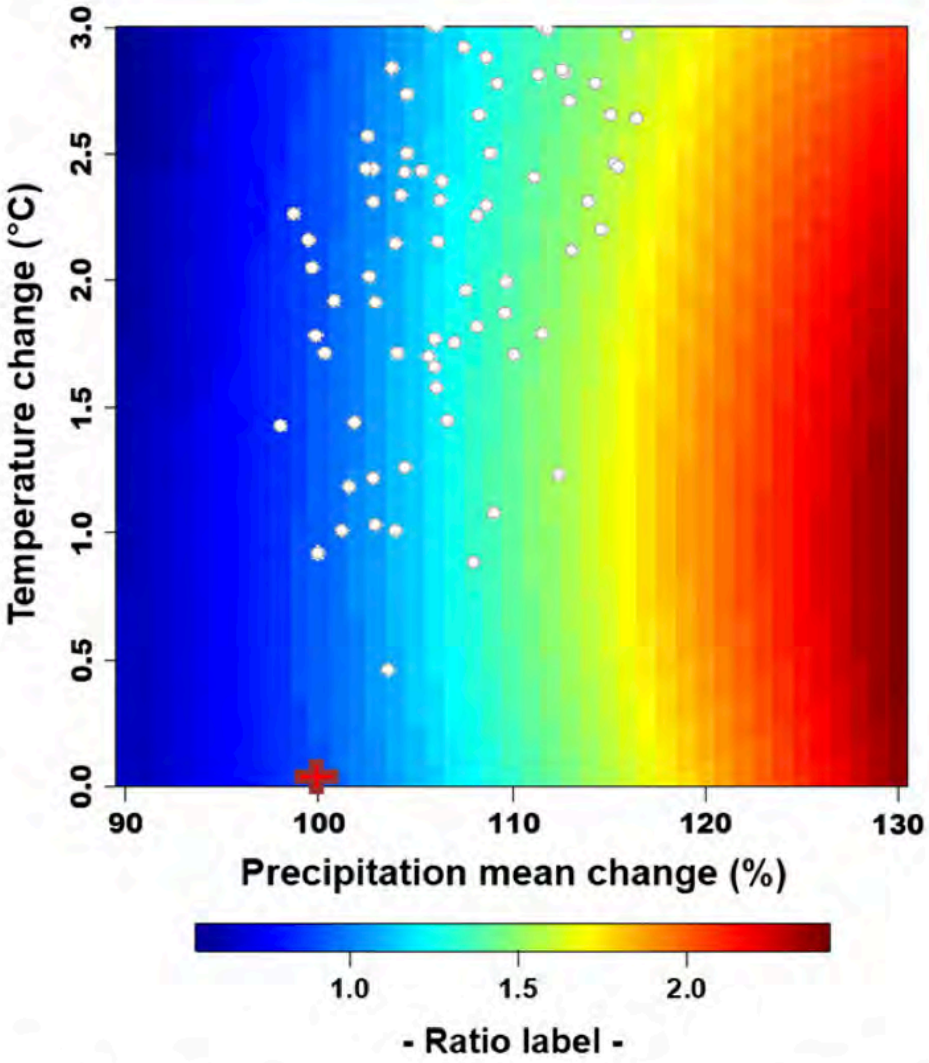


FIGURE 9 Climate response surfaces of the ratio of $\overline{SSC_{max}}$ between M_6 for a given climate scenario and the value of $\overline{SSC_{max}}$ under historical conditions (red cross). The responses are averaged over 50 simulations. Coupled Model Intercomparison Project Phase 5 precipitation and temperature projections (white circles) are also shown